

Measuring $\sin 2\beta$ in $B_s^0(t) \rightarrow \phi K_s$

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(February 1, 2008)

Abstract

We show that, unlike other pure $b \rightarrow d$ penguin processes, the decay $B_s^0(t) \rightarrow \phi K_s$ is dominated by a single amplitude, that of the internal t -quark. The contributions of the u - and c -quark operators each vanish due to a cancellation between the $(V - A) \otimes (V - A)$ and $(V - A) \otimes (V + A)$ matrix elements. Thus, the indirect CP asymmetry in this decay probes $\sin 2\beta$. Although this cancellation is complete only for certain values of the s - and b -quark masses, the theoretical uncertainty on $\sin 2\beta$ is still less than 10% over most ($\sim 80\%$) of the parameter space. By measuring the direct CP asymmetry, one can get a better idea of the probable error on $\sin 2\beta$.

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It has been known for many years that the B system is a particularly good place to test the standard model (SM) explanation of CP violation. By measuring CP-violating rate asymmetries in the decays of neutral B mesons to a variety of final states, one can cleanly extract the CP phases α , β and γ [1]. This allows one to construct the unitarity triangle [2] and search for the presence of physics beyond the SM.

In the early days of the field, only tree-level decays of B mesons were considered. However, it was soon realized that penguin amplitudes could play an important role [3, 4]. For example, the presence of penguin contributions in $B_d^0(t) \rightarrow \pi^+\pi^-$ can spoil the clean extraction of α (though this can be rectified with the help of an isospin analysis [5]). And the clean measurement of γ via $B_s^0(t) \rightarrow \rho K_s$ is completely ruined since, for this decay, the penguin amplitude is the dominant contribution.

Given the importance of such penguin contributions, one is immediately led to consider CP violation in pure penguin decays. In the (approximate) Wolfenstein parametrization [6] of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, there are only two matrix elements which have a nonzero weak phase: $V_{td} \propto \exp(-i\beta)$ and $V_{ub} \propto \exp(-i\gamma)$. Thus, assuming that the penguin amplitudes are dominated by an internal t -quark, one expects that the $b \rightarrow s$ penguin amplitude, which involves the product of CKM matrix elements $V_{tb}V_{ts}^*$, is real, to a good approximation. Similarly, the weak phase of the $b \rightarrow d$ penguin amplitude ($V_{tb}V_{td}^*$) is $+\beta$. Knowing that the weak phases of $B_d^0\text{-}\overline{B}_d^0$ and $B_s^0\text{-}\overline{B}_s^0$ mixing are, respectively, $-\beta$ and 0, this allows us to compute the weak phase probed in various pure-penguin decay asymmetries [3]:

$$b \rightarrow d \quad : \quad \text{Asym}(B_d^0(t) \rightarrow K^0\overline{K}^0) \sim 0 \quad , \quad (1)$$

$$\text{Asym}(B_s^0(t) \rightarrow \phi K_s) \sim -\sin 2\beta \quad , \quad (2)$$

$$b \rightarrow s \quad : \quad \text{Asym}(B_d^0(t) \rightarrow \phi K_s) \sim +\sin 2\beta \quad , \quad (3)$$

$$\text{Asym}(B_s^0(t) \rightarrow \phi\phi) \sim 0 \quad . \quad (4)$$

The problem with the above analysis is that the $b \rightarrow d$ penguin amplitude is *not* dominated by an internal t -quark. In the quark-level decays $b \rightarrow u\bar{u}d$ and $b \rightarrow c\bar{c}d$, the $u\bar{u}$ and $c\bar{c}$ quark pairs can rescatter strongly into an $s\bar{s}$ quark pair, giving effective $V_{ub}V_{ud}^*$ and $V_{cb}V_{cd}^*$ contributions to the $b \rightarrow d$ penguin decays above. Buras and Fleischer have estimated that these contributions can be between 20% and 50% of the leading t -quark contribution [7]. And since the u - and c -quark contributions have a different weak phase than that of the t -quark contribution, this implies that the weak phase of the $b \rightarrow d$ penguin amplitude is *not* $+\beta$, so that the predictions of Eqs. (1) and (2) are not valid. On the contrary, due to the presence of these several decay amplitudes, one expects that a weak phase *cannot* be cleanly extracted from the measurement of CP asymmetries in pure $b \rightarrow d$ penguin decays. One also expects to observe direct CP violation in such decays.

Note that these same conclusions do not hold for $b \rightarrow s$ penguin amplitudes. In this case, the CKM matrix-element product associated with the u -quark contribution, $V_{ub}V_{us}^*$, is a small fraction ($\sim 2\%$) of those of the corresponding c - and t -quark contributions, $V_{cb}V_{cs}^*$ and $V_{tb}V_{ts}^*$. Since the c - and t -quark CKM matrix elements are both real, the assumption that the $b \rightarrow s$ penguin amplitude is real is a good approximation. Thus, the predictions of Eqs. (3) and (4) still hold.

In this paper, we re-examine the question of the weak phase of the $b \rightarrow d$ penguin for the exclusive decay $B_s^0(t) \rightarrow \phi K_s$ [Eq. (2)]. As we will show, although the quark-level contributions from u - and c -quarks are non-negligible, at the meson level the matrix elements involving the corresponding u - and c -quark operators each vanish, to a good approximation, over a large region of parameter space. This is due to two factors. First, for a large range of values of the s - and b -quark masses, it is a fortuitous numerical coincidence that the matrix element of the $(V - A) \otimes (V - A)$ piece of the u/c -quark operator is approximately equal to that of the $(V - A) \otimes (V + A)$ piece. Second, because the final state consists of a vector meson and a pseudoscalar meson, the full matrix element is proportional to the difference between these two pieces. In other words, there is a cancellation between the $(V - A) \otimes (V - A)$ and $(V - A) \otimes (V + A)$ matrix elements. Thus, to the extent that this cancellation is complete, the CP-violating rate asymmetry in $B_s^0(t) \rightarrow \phi K_s$ still cleanly probes the weak phase β .

We begin the discussion by considering the SM effective hamiltonian for hadronic B decays of the type $b \rightarrow dff$ [8]:

$$H_{eff}^{b \rightarrow d} = \frac{G_F}{\sqrt{2}} [V_{fb} V_{fd}^* (c_1 O_{1f}^d + c_2 O_{2f}^d) - \sum_{i=3}^{10} (V_{ub} V_{ud}^* c_i^u + V_{cb} V_{cd}^* c_i^c + V_{tb} V_{td}^* c_i^t) O_i^d] + h.c. \quad (5)$$

In the first two terms, f can be a u or a c quark, while in the last three terms, the superscript u , c or t indicates the flavour of the internal quark. The operators O_i^d are defined as

$$\begin{aligned} O_{1f}^d &= \bar{d}_\alpha \gamma_\mu L f_\beta \bar{f}_\beta \gamma^\mu L b_\alpha & O_{2f}^d &= \bar{d}_\alpha \gamma_\mu L f \bar{f} \gamma^\mu L b \\ O_{3,5}^d &= \bar{d}_\alpha \gamma_\mu L b \bar{q}' \gamma_\mu L(R) q' & O_{4,6}^d &= \bar{d}_\alpha \gamma_\mu L b_\beta \bar{q}'_\beta \gamma_\mu L(R) q'_\alpha \\ O_{7,9}^d &= \frac{3}{2} \bar{d}_\alpha \gamma_\mu L b e_{q'} \bar{q}' \gamma^\mu R(L) q' & O_{8,10}^d &= \frac{3}{2} \bar{d}_\alpha \gamma_\mu L b_\beta e_{q'} \bar{q}'_\beta \gamma_\mu R(L) q'_\alpha, \end{aligned} \quad (6)$$

where $R(L) = 1 \pm \gamma_5$, and q' is summed over u , d and s . O_1 and O_2 are, respectively, the direct and QCD-corrected tree-level operators, O_{3-6} are the strong gluon-induced penguin operators, and operators O_{7-10} are due to γ and Z exchange (electroweak penguins), and “box” diagrams at loop level. The Wilson coefficients c_i^f are defined at the scale $\mu \approx m_b$ and have been evaluated to next-to-leading order in QCD. Below we give the regularization-scheme-independent values for the c_i^f for $m_t = 176$ GeV, $\alpha_s(m_Z) = 0.117$, and $\mu = m_b = 5$ GeV [9]:

$$\begin{aligned} c_1 &= -0.307, \quad c_2 = 1.147 \\ c_3^t &= 0.017, \quad c_4^t = -0.037, \quad c_5^t = 0.010, \quad c_6^t = -0.045, \\ c_7^t &= -1.24 \times 10^{-5}, \quad c_8^t = 3.77 \times 10^{-4}, \quad c_9^t = -0.010, \quad c_{10}^t = 2.06 \times 10^{-3}, \\ c_{3,5}^i &= -c_{4,6}^i / N_c = P_s^i / N_c, \quad c_{7,9}^i = P_e^i, \quad c_{8,10}^i = 0, \quad i = u, c, \end{aligned} \quad (7)$$

where N_c is the number of colors. The leading contributions to $P_{s,e}^i$ are given by $P_s^i = (\frac{\alpha_s}{8\pi}) c_2 (\frac{10}{9} + G(m_i, \mu, q^2))$ and $P_e^i = (\frac{\alpha_{em}}{9\pi}) (N_c c_1 + c_2) (\frac{10}{9} + G(m_i, \mu, q^2))$, where

the function $G(m, \mu, q^2)$ takes the form

$$G(m, \mu, q^2) = 4 \int_0^1 x(1-x) \ln \frac{m^2 - x(1-x)q^2}{\mu^2} dx . \quad (8)$$

All the above coefficients are obtained up to one-loop order in the electroweak interactions. The momentum q is the momentum carried by the virtual gluon in the penguin diagram. Note that $c_4^i = c_6^i$, $i = u, c$, while $c_4^t \neq c_6^t$. This will be important in what follows.

For the decay $B \rightarrow f_1 f_2$, we are really interested in the matrix elements of the various operators. We therefore define new coefficients $\bar{c}_i^{u,c}$ as

$$\bar{c}_i^{u,c} = \frac{\langle f_1 f_2 | c_i^{u,c}(q^2) O_i | B \rangle}{\langle f_1 f_2 | O_i | B \rangle} . \quad (9)$$

From Eq. (7), the $\bar{c}_i^{u,c}$ can be expressed in terms of the function $\bar{G}_i^{u,c}$ defined as

$$\bar{G}_i^{u,c}(m_{u,c}, \mu) = \frac{\langle f_1 f_2 | G(m_{u,c}, \mu, q^2) O_i | B \rangle}{\langle f_1 f_2 | O_i | B \rangle} . \quad (10)$$

One can use models to calculate the functions $\bar{G}_i^{u,c}$, and one finds in general that the functions $\bar{G}_i^{u,c}$ are process dependent. More importantly, it is a reasonable assumption that the functions $\bar{G}_i^{u,c}$ are independent of i :

$$\bar{G}_i^{u,c}(m_{u,c}, \mu) = \bar{G}_j^{u,c}(m_{u,c}, \mu) . \quad (11)$$

This is because the effects of the different Dirac structures of the operators O_i cancel in the ratio in Eq. (10) [10]. This implies that the relations between the various $\bar{c}_i^{u,c}$ are the same as those between the various $c_i^{u,c}(q^2)$. In particular, we have

$$\bar{c}_4^i = \bar{c}_6^i, i = u, c . \quad (12)$$

This will be the key ingredient in the analysis below. In the study of non-leptonic decays the usual practice is to replace

$$\bar{c}_i^{u,c} \rightarrow c_i^{u,c}(q_{av}^2) , \quad (13)$$

where q_{av}^2 is allowed to vary between $m_b^2/4$ and $m_b^2/2$ to account for process dependence [11, 12].

The structure of the effective Hamiltonian allows us to write the amplitude for $\overline{B}_s^0 \rightarrow \phi K_s$ as

$$A_s^{\phi K_s} \equiv A(\overline{B}_s^0 \rightarrow \phi K_s) = \frac{G_F}{\sqrt{2}} (V_{ub} V_{ud}^* P_u + V_{cb} V_{cd}^* P_c + V_{tb} V_{td}^* P_t) . \quad (14)$$

Under the assumption of naive factorization one can write, dropping factors common to $P_{u,c,t}$, and using the fact that $\bar{c}_6^i = \bar{c}_4^i$, $i = u, c$ [Eq. (12)],

$$P_{u,c} = \bar{c}_6^{u,c} \left(1 - \frac{1}{N_c^2}\right) [\langle O_{LL} \rangle - 2 \langle O_{SP} \rangle] , \quad (15)$$

where

$$\begin{aligned}\langle O_{LL} \rangle &= \langle \phi | \bar{s} \gamma_\mu (1 - \gamma_5) b | \overline{B_s^0} \rangle \langle K_s | \bar{d} \gamma^\mu (1 - \gamma_5) s | 0 \rangle , \\ \langle O_{SP} \rangle &= \langle \phi | \bar{s} (1 - \gamma_5) b | \overline{B_s^0} \rangle \langle K_s | \bar{d} (1 + \gamma_5) s | 0 \rangle .\end{aligned}\quad (16)$$

(The operator O_{SP} appears due to a Fierz transformation: $(V - A) \otimes (V + A) = -2(S - P) \otimes (S + P)$.) On the other hand, the contribution from the top penguin is more complicated:

$$\begin{aligned}P_t &= \left[(c_4^t + \frac{c_3^t}{N_c}) \langle O_{LL} \rangle + (c_3^t + \frac{c_4^t}{N_c}) \langle O_{LL1} \rangle \right] \\ &+ \left[-2(c_6^t + \frac{c_5^t}{N_c}) \langle O_{SP} \rangle + (c_5^t + \frac{c_6^t}{N_c}) \langle O_{LR1} \rangle \right] \\ &- \frac{1}{2} \left[(c_9^t + \frac{c_{10}^t}{N_c}) \langle O_{LL1} \rangle + (c_{10}^t + \frac{c_9^t}{N_c}) \langle O_{LL} \rangle \right] ,\end{aligned}\quad (17)$$

where

$$\begin{aligned}\langle O_{LL1} \rangle &= \langle \phi | \bar{s} \gamma_\mu (1 - \gamma_5) s | 0 \rangle \langle K_s | \bar{d} \gamma^\mu (1 - \gamma_5) b | \overline{B_s^0} \rangle , \\ \langle O_{LR1} \rangle &= \langle \phi | \bar{s} \gamma_\mu (1 + \gamma_5) s | 0 \rangle \langle K_s | \bar{d} \gamma^\mu (1 - \gamma_5) b | \overline{B_s^0} \rangle .\end{aligned}\quad (18)$$

In the above, we have neglected the contributions from $c_{7,8}$.

It is convenient to rewrite $P_{u,c}$ and P_t as

$$\begin{aligned}P_{u,c} &= \bar{c}_6^{u,c} (1 - \frac{1}{N_c^2}) X \langle O_{LL} \rangle , \\ P_t &= a_6 X \langle O_{LL} \rangle + (a_4 - a_6 - \frac{1}{2} a_{10}) \langle O_{LL} \rangle + (a_3 + a_5 - \frac{1}{2} a_9) \langle O_{LL1} \rangle ,\end{aligned}\quad (19)$$

where $\langle O_{LL1} \rangle = \langle O_{LR1} \rangle$,

$$a_i = \begin{cases} c_i + \frac{c_{i-1}}{N_c} , & i = 4, 6, 10 , \\ c_i + \frac{c_{i+1}}{N_c} , & i = 3, 5, 9 , \end{cases}\quad (20)$$

and

$$X \equiv \left[1 - \frac{2 \langle O_{SP} \rangle}{\langle O_{LL} \rangle} \right] .\quad (21)$$

It is this latter quantity X which is the focus of our attention in this paper.

Using the fact that

$$\langle K_s(p_K) | \bar{d} \gamma_\mu (1 - \gamma_5) s | 0 \rangle = i f_{K_S} p_{K\mu} ,\quad (22)$$

along with the equations of motion for the quarks (we assume that $p_K = p_d + p_{\bar{s}}$), it is straightforward to show that

$$X = \left[1 - 2 \frac{1}{m_b + m_s} \frac{m_K^2}{m_s + m_d} \right] .\quad (23)$$

However, the key point is the following: taking $m_K = 500$ MeV, $m_b = 4.9$ GeV, $m_s = 100$ MeV (all at the b -quark mass scale), and $m_d \simeq 0$, one finds that $X = 0$! Thus, the matrix elements vanish for u and c but do *not* vanish for t . The decay $B_s^0(t) \rightarrow \phi K_S$ is therefore dominated by a single decay amplitude — the t -quark penguin contribution — and a measurement of the CP-violating rate asymmetry probes the angle β [Eq. (2)].

This result is related specifically to two facts: (i) the decay is a pure penguin decay, and (ii) the final state ϕK_S consists of a vector and a pseudoscalar meson. This can be seen as follows. First, consider a decay such as $B_d^0(t) \rightarrow \pi^+\pi^-$, which receives a tree-level contribution $TV_{ub}V_{ud}^*\mathcal{O}_T$, where the operator \mathcal{O}_T is of the form $(V-A)\otimes(V-A)$. As we saw above, the matrix element of this operator alone does not vanish. Thus, one must consider decays which have no tree-level contributions. That is, only processes involving the quark-level transition $b \rightarrow ds\bar{s}$ need be considered. Now consider a pure-penguin final state containing two pseudoscalars, such as $B_d^0(t) \rightarrow K^0\bar{K}^0$. Repeating the above analysis, one finds that

$$X = \left[1 + 2 \frac{1}{m_b - m_s} \frac{m_K^2}{m_s + m_d} \right]. \quad (24)$$

Clearly there is now no cancellation between the two contributions, and one finds $X \neq 0$. Similarly, if the final state consisted of two vector mesons, such as in $B_d^0(t) \rightarrow K^{*0}\bar{K}^{*0}$, then the $\langle O_{SP} \rangle$ matrix element would vanish due to conservation of angular momentum, and again we would find $X \neq 0$. Thus, only final states which consist of a vector and a pseudoscalar can have $X = 0$.

In fact, $B_s^0(t) \rightarrow \phi K_S$ is the *only* decay involving a $b \rightarrow d$ penguin amplitude for which the \mathcal{O}_u and \mathcal{O}_c matrix elements vanish. The only other $b \rightarrow ds\bar{s}$ decay whose final state consists of a vector and a pseudoscalar is $B_d^0(t) \rightarrow \bar{K}^0 K^{0*}$. However, $B_d^0 \rightarrow \bar{K}^0 K^{0*}$ and $\bar{B}_d^0 \rightarrow \bar{K}^0 K^{0*}$ do not factorize in the same way. Specifically, for $B_d^0 \rightarrow \bar{K}^0 K^{0*}$, the \bar{K}^0 is coupled to the vacuum as in Eq. (16), so that $X = 0$. However, for $\bar{B}_d^0 \rightarrow \bar{K}^0 K^{0*}$, it is the K^{0*} which is coupled to the vacuum, which implies that the $\langle O_{SP} \rangle$ matrix element vanishes. Thus, $X \neq 0$ for this decay. Therefore it is only the decay $B_s^0(t) \rightarrow \phi K_S$ whose indirect CP asymmetry is expected to measure $\sin 2\beta$.

Now, the vanishing of the u - and c -quark matrix elements in $B_s^0(t) \rightarrow \phi K_S$ depends on two theoretical ingredients: (i) we have assumed that naive factorization is valid for this decay, and (ii) we have taken particular values for m_s and m_b . It is important to examine the extent to which these assumptions are justified, and to show how the result changes when these assumptions are relaxed.

First of all, we can see the effect of a nonzero value of X as follows. The measurement of the time-dependent rate $B_s^0(t) \rightarrow \phi K_S$ allows one to extract both direct and indirect CP-violating asymmetries. These are defined as follows:

$$\begin{aligned} a_{dir}^{CP} &= \frac{|A_s^{\phi K_S}|^2 - |\bar{A}_s^{\phi K_S}|^2}{|A_s^{\phi K_S}|^2 + |\bar{A}_s^{\phi K_S}|^2}, \\ a_{indir}^{CP} &= \frac{\text{Im} \left(A_s^{\phi K_S^*} \bar{A}_s^{\phi K_S} \right)}{|A_s^{\phi K_S}|^2 + |\bar{A}_s^{\phi K_S}|^2}, \end{aligned} \quad (25)$$

where we have assumed that the weak phase of $B_s^0-\overline{B}_s^0$ mixing is negligible, as is the case in the SM. $A_s^{\phi K_S}$ is defined in Eq. (14), and $\overline{A}_s^{\phi K_S}$ is obtained from $A_s^{\phi K_S}$ by changing the signs of the weak phases. Using CKM unitarity to eliminate the $V_{ub}V_{ud}^*$ term in Eq. (14), $A_s^{\phi K_S}$ can be written as

$$A_s^{\phi K_S} = \frac{G_F}{\sqrt{2}}(\mathcal{P}_{cu}e^{i\delta_c} + \mathcal{P}_{tu}e^{i\delta_t}e^{-i\beta}) , \quad (26)$$

where we have explicitly separated out the strong phases δ_c and δ_t , as well as the weak phase β . The magnitudes of the CKM matrix elements have been absorbed into the definitions of \mathcal{P}_{cu} and \mathcal{P}_{tu} . Using this expression for $A_s^{\phi K_S}$, the CP asymmetries take the form

$$\begin{aligned} a_{dir}^{CP} &= \frac{2\mathcal{P}_{cu}\mathcal{P}_{tu}\sin\beta\sin\Delta}{\mathcal{P}_{tu}^2 + \mathcal{P}_{cu}^2 + 2\mathcal{P}_{tu}\mathcal{P}_{cu}\cos\beta\cos\Delta} , \\ a_{indir}^{CP} &= \frac{\mathcal{P}_{tu}^2\sin 2\beta + 2\mathcal{P}_{cu}\mathcal{P}_{tu}\sin\beta\cos\Delta}{\mathcal{P}_{tu}^2 + \mathcal{P}_{cu}^2 + 2\mathcal{P}_{tu}\mathcal{P}_{cu}\cos\beta\cos\Delta} , \end{aligned} \quad (27)$$

where $\Delta \equiv \delta_t - \delta_c$. From these expressions, we see that a nonzero value of X corresponds to a nonzero value of \mathcal{P}_{cu} . This in turn leads to a nonzero value of the direct CP asymmetry a_{dir}^{CP} , and also affects the clean extraction of $\sin 2\beta$ from the indirect CP asymmetry. In order to compute the error on the measurement of $\sin 2\beta$, we will need to estimate the size of the ratio $\mathcal{P}_{cu}/\mathcal{P}_{tu}$, as well as the strong phase Δ .

The main factor which may contribute to $X \neq 0$ is if the masses m_b and m_s do not take the respective values 4.9 GeV and 100 MeV. In computing the CP asymmetries in Eq. (27), we will allow m_b and m_s to take a range of values. In our calculation, we use current quark masses, evaluated at the scale $\mu \sim m_b$. For the b -quark mass, we take $4.35 \leq m_b \leq 4.95$ GeV. As for the current strange-quark mass, there is a great deal of uncertainty in the value of m_s . One can obtain $m_s(\mu = m_b)$ by using as input $m_s(\mu = 1 \text{ GeV})$ and then using QCD running to increase the scale up to $\mu = m_b$. However, $m_s(\mu = 1 \text{ GeV})$ is not very well known and can range from 0.150 to 0.200 GeV [13]. In addition, it is not clear that perturbative calculations are reliable near $\mu \sim 1 \text{ GeV}$. Given all these uncertainties, we vary $m_s(m_b)$ in the range $0.08 \leq m_s \leq 0.12 \text{ GeV}$.

There are also nonfactorizable effects which might give rise to $X \neq 0$. There have been several attempts to calculate corrections to the naive factorization assumption. One promising approach is QCD-improved factorization [14], in which one systematically calculates corrections to naive factorization in an expansion in $\alpha_s(m_b) \sim 0.2$ and Λ_{QCD}/m_b . Naive factorization appears as the leading-order term in this expansion. If we consider QCD corrections to this term, we note that the $P_{u,c}$ arise already at $O(\alpha_s)$, and so they receive no corrections at this order. In fact, the $P_{u,c}$ are part of the $O(\alpha_s)$ corrections to the naive factorization results. There are additional α_s corrections which can be taken into account by the replacement $a_i \rightarrow a_i^{eff} = a_i(1 + r_i)$ in Eq. (19), where $r_i \sim O(\alpha_s)$ are process-dependent corrections to the naive factorization assumption.

Corrections to the $P_{u,c}$ in Eq. (15) may then arise if Eq. (11) does not hold. One

can then write

$$P_{u,c} = (1 - \frac{1}{N_c^2}) [\bar{c}_4^{u,c} \langle O_{LL} \rangle - 2\bar{c}_6^{u,c} \langle O_{SP} \rangle] = \bar{c}_6^{u,c} (1 - \frac{1}{N_c^2}) [X + X_1] \langle O_{LL} \rangle , \quad (28)$$

where

$$X_1 = \frac{\bar{c}_4^{u,c} - \bar{c}_6^{u,c}}{\bar{c}_6^{u,c}} . \quad (29)$$

Note that X_1 is complex, so that if one writes $X_1 = |X_1|e^{i\theta}$, one obtains

$$|P_{u,c}| = |\bar{c}_6^{u,c}| (1 - \frac{1}{N_c^2}) \sqrt{X^2 + X_1^2 + 2|X||X_1| \cos \theta} \langle O_{LL} \rangle . \quad (30)$$

In what follows we assume that the effect of X_1 is small enough to be absorbed in the uncertainty in the quark masses in the expression for X in Eq. (23). In other words the effect of X_1 is essentially taken into account by varying the quark masses in X .

In order to test the robustness of the claim that the indirect CP asymmetry in $B_d^0(t) \rightarrow \phi K_S$ measures $\sin 2\beta$, we perform the following analysis. We scan the entire parameter space, calculating the CP asymmetries a_{dir}^{CP} and a_{indir}^{CP} of Eq. (27) at each point in this space. We are especially interested in the quantity δ , which measures the fractional difference between the indirect CP asymmetry and the true value of $\sin 2\beta$:

$$\delta \equiv \frac{a_{indir}^{CP} - \sin 2\beta}{\sin 2\beta} . \quad (31)$$

In particular, we wish to compute what fraction of the parameter space leads to a given value for δ . This will give us some sense of the extent to which the asymmetry in $B_d^0(t) \rightarrow \phi K_S$ truly probes β .

In addition to the masses m_b and m_s , the CP asymmetries depend on other quantities. First, they depend on the momentum transfer q_{av}^2 , which we vary between $m_b^2/4$ and $m_b^2/2$. Second, they involve the CKM parameters ρ and η , whose ranges are taken from Ref. [15]. Third, one also needs values for the d - and c -quark masses. We take $m_d(\mu = m_b) = 6$ MeV and $1.1 \leq m_c \leq 1.4$ GeV [2]. Finally, note that, while $P_{u,c}$ depend only on the matrix element $\langle O_{LL} \rangle$, P_t also depends on $\langle O_{LL1} \rangle$ [Eq. (19)]. Using factorization, one can write

$$r' = \frac{\langle O_{LL1} \rangle}{\langle O_{LL} \rangle} = \frac{f_\phi F_1^{K_S}(m_\phi^2)}{f_{K_S} A_0^\phi(m_K^2)} , \quad (32)$$

where the various formfactors and decay constants above are defined by

$$\begin{aligned} \langle K_S(q) | \bar{d} \gamma_\mu (1 - \gamma_5) u | 0 \rangle &= i f_{K_S} q_\mu , \\ \langle \phi(q, \epsilon) | \bar{s} \gamma_\mu \gamma_5 b | \overline{B_d^0}(p) \rangle &= (M_B + M_\phi) A_1^\phi \left[\epsilon_\mu^* - \frac{\epsilon^* \cdot (p - q)}{(p - q)^2} (p - q)_\mu \right] \\ &\quad - A_2^\phi \frac{\epsilon^* \cdot (p - q)}{M_B + M_\phi} \left[(P_B + P_\phi)_\mu - \frac{M_B^2 - M_\phi^2}{(p - q)^2} (p - q)_\mu \right] \end{aligned}$$

$$\begin{aligned}
& +2M_\phi A_0^\phi \frac{\epsilon^* \cdot (p-q)}{(p-q)^2} (p-q)_\mu , \\
\langle K_S(q) | \bar{u} \gamma_\mu b | \overline{B_d^0}(p) \rangle &= F_1^{K_S} \left[(p+q)^\mu - \frac{m_B^2 - m_{K_S}^2}{(p-q)^2} (p-q)^\mu \right] \\
& + F_0^{K_S} \frac{m_B^2 - m_{K_S}^2}{(p-q)^2} (p-q)^\mu , \\
\langle 0 | \bar{s} \gamma^\mu s | \phi(q, \epsilon) \rangle &= M_\phi f_\phi \epsilon^\mu .
\end{aligned} \tag{33}$$

Using $f_\phi = 0.237$ GeV [16] and the various form factors calculated using the light cone sum rules [17] we obtain $r' \approx 1.5$.

We are now in a position to calculate the CP asymmetries a_{dir}^{CP} and a_{indir}^{CP} in $B_s^0(t) \rightarrow \phi K_S$ [Eq. (27)], as well as the deviation of the indirect CP asymmetry from the true value of $\sin 2\beta$. In Fig. 1, we plot the fraction of the parameter space for which $|\delta|$, the error on $\sin 2\beta$ as extracted from the indirect CP asymmetry [Eq. (31)], is less than a given value, $|\delta|_{max}$. From this figure, we see that $\sin 2\beta$ can be obtained with an error less than 30% over virtually the entire parameter space. And this error is reduced to about 10% in 80% of the parameter space. While this should not be interpreted statistically as some sort of confidence level, it does indicate that it is quite likely that β can be extracted from the indirect CP asymmetry in $B_s^0(t) \rightarrow \phi K_S$ with a rather small error.

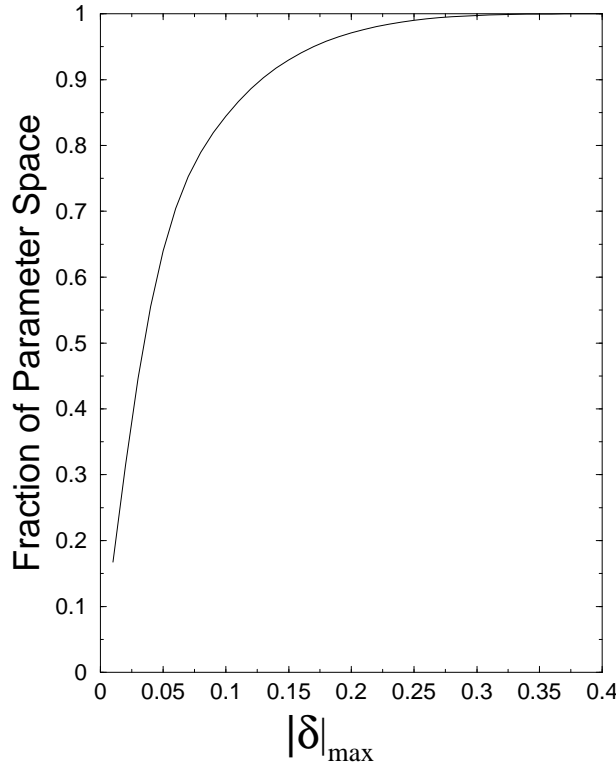


Figure 1: The fraction of the parameter space for which the error $|\delta|$ in extracting $\sin 2\beta$ from a_{indir}^{CP} is less than a given value, $|\delta|_{max}$.

Of course, if the time-dependent rate for $B_s^0(t) \rightarrow \phi K_s$ is measured, we will have more information than just a_{indir}^{CP} : we will also measure the direct CP asymmetry a_{dir}^{CP} . Since a_{dir}^{CP} vanishes if $X = 0$, its value may help us determine the extent to which a_{indir}^{CP} really measures $\sin 2\beta$. At first glance, the correlation between a_{indir}^{CP} and a_{dir}^{CP} appears airtight: if a_{dir}^{CP} is found to vanish, then this must imply that $X = 0$, so that a_{indir}^{CP} yields $\sin 2\beta$. Unfortunately, things are not quite so straightforward: a_{dir}^{CP} is also proportional to the strong phase difference Δ [Eq. (27)]. Therefore, if $\Delta \simeq 0$, then a_{dir}^{CP} will vanish even if $X \neq 0$. Thus, this possibility must be taken into account in evaluating the correlation between the measurements of a_{indir}^{CP} and a_{dir}^{CP} .

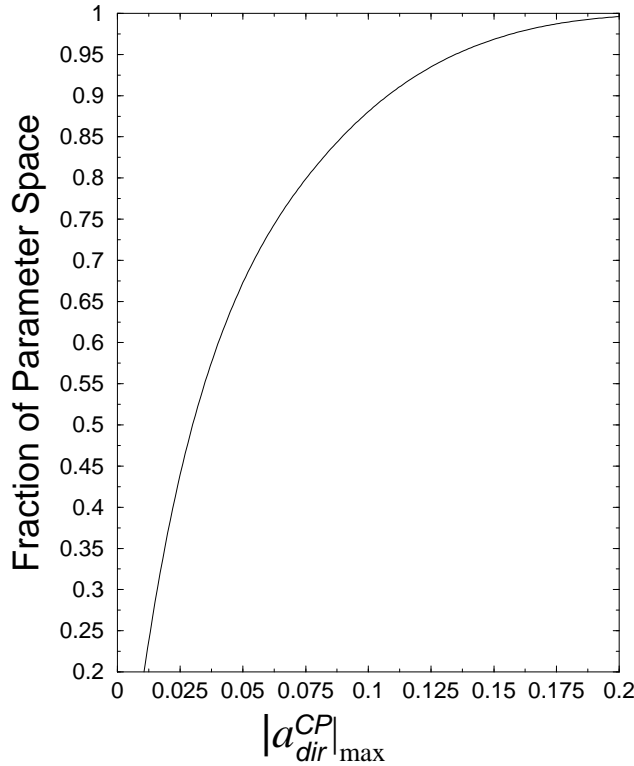


Figure 2: The fraction of the parameter space for which a_{dir}^{CP} is less than a given value, $|a_{dir}^{CP}|_{\max}$.

In Fig. 2, we plot the fraction of the parameter space for which a_{dir}^{CP} is less than a given maximal value. As is clear from the figure, we expect a_{dir}^{CP} to be at most 20% – larger values would point to the presence of new physics. Fig. 3 shows, for a given value of $|a_{dir}^{CP}|$, the fraction of the parameter space for which $|\delta|$ is less than a particular maximum error ($|\delta|_{\max} = 5\%, 10\%, 15\%, 20\%$). From this plot, we see that if a_{dir}^{CP} is measured to be 0.1, one can obtain $\sin 2\beta$ from a_{indir}^{CP} with an error of 5% (20%) in $\sim 55\%$ ($\sim 95\%$) of the parameter space. If a_{dir}^{CP} is found to be tiny, then this is probably due to the fact that $X \simeq 0$, since $\delta < 5\%$ over $\sim 90\%$ of the parameter space. However, as discussed above, this does not hold over the entire space since a_{dir}^{CP} can be small if $\Delta \simeq 0$, while $X \neq 0$.

There is one technical point which is worth mentioning here. Naively, one would

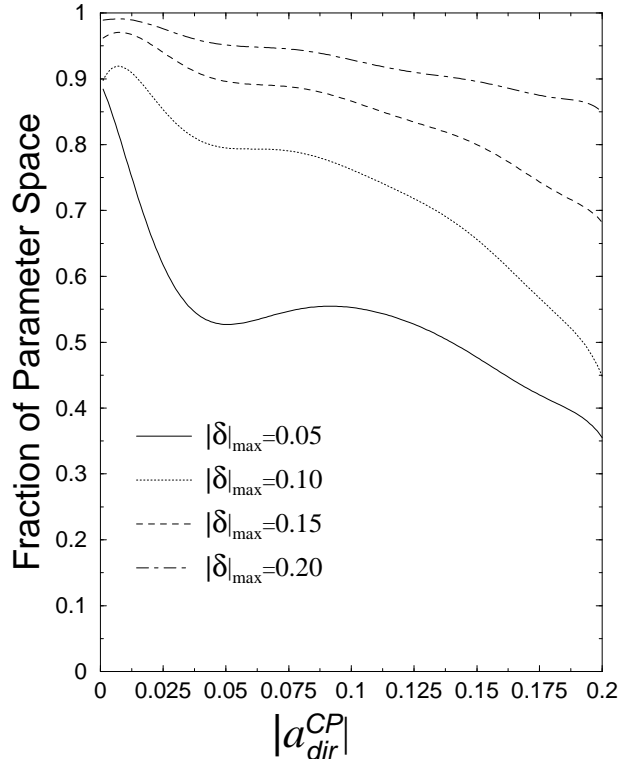


Figure 3: The fraction of the parameter space for which δ can be measured with a maximum error of $|\delta|_{max} = 0.05, 0.1, 0.15$ and 0.20 , as a function of $|a_{dir}^{CP}|$.

expect that, as a_{dir}^{CP} increases, the error δ will also increase. That is, one expects that the fraction of the parameter space with a given maximum error $|\delta|_{max}$ should decrease with increasing a_{dir}^{CP} . This is because the prediction that the indirect CP asymmetry in $B_s^0(t) \rightarrow \phi K_S$ measures $\sin 2\beta$ depends on the fact that $X = 0$, and a_{dir}^{CP} increases with increasing X . However, Fig. 3 does not behave in exactly this way: although the fraction of the parameter space with a given $|\delta|_{max}$ does indeed roughly decrease with increasing a_{dir}^{CP} , this decrease is not monotonic. For example, the curve for $|\delta|_{max} = 0.05$ flattens out at around $|a_{dir}^{CP}| \sim 0.04$, and even turns up before falling again for $|a_{dir}^{CP}| \gtrsim 0.1$. The explanation for this behaviour is as follows. From Eq. (27), we see that $|a_{dir}^{CP}|$ depends, among other things, on the strong phase difference Δ and the ratio $r = \mathcal{P}_{cu}/\mathcal{P}_{tu}$. As q_{av}^2 is varied, keeping other parameters fixed, we find that, as r increases, so does $\sin \Delta$. However, $|\delta|$ depends on $\cos \Delta$, which obviously decreases as $\sin \Delta$ increases. Thus, there is a region of parameter space where, as r and $\sin \Delta$ increase, the error $|\delta|$ actually decreases due to the presence of the $\cos \Delta$ term. It is this effect which is responsible for the flattening out and the slight rise in the curves in Fig. 3.

Finally, one can take a most conservative point of view, and ask what is the maximum error $|\delta|_{max}$ over the entire parameter space for a given measurement of $|a_{dir}^{CP}|$. This is shown in Fig. 4. Depending on the value of $|a_{dir}^{CP}|$, $|\delta|_{max}$ is between 30 and 40%. As was the case in Fig. 3, the downturn of the curve as $|a_{dir}^{CP}|$ increases

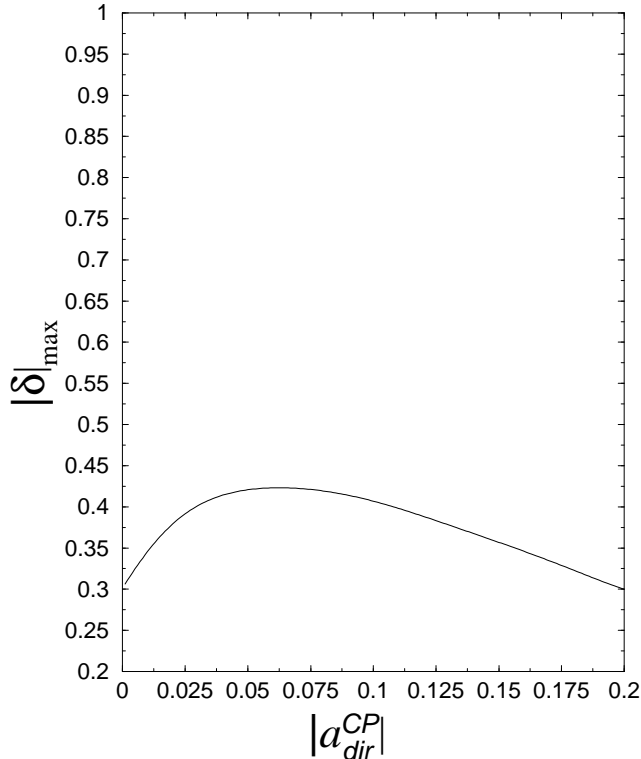


Figure 4: The maximum error $|\delta|_{max}$ for the entire parameter space as a function of $|a_{dir}^{CP}|$.

is due to the dependence of δ on $\cos \Delta$.

Of course, if X does indeed vanish, this may have some negative practical implications. Specifically, since there are fewer contributions to the amplitude for $B_s^0 \rightarrow \phi K_s$, one might suspect that the branching ratio will be smaller than that of other pure $b \rightarrow d$ penguin decays. This is indeed what we find: taking $\rho = 0.18$ and $\eta = 0.36$ [15], $P_u = P_c = 0$, and using the form factors in Ref [17], we obtain

$$\frac{BR[B_s^0 \rightarrow \phi K_s]}{BR[\overline{B}_d^0 \rightarrow \rho^+ \pi^-]} = 0.003 . \quad (34)$$

Using the measured $BR[\overline{B}_d^0 \rightarrow \rho^+ \pi^-] = 30 \times 10^{-6}$ [18] we find $BR[B_s^0 \rightarrow \phi K_s] \sim 10^{-7}$, which is very small.

Fortunately, the above analysis for $B_s^0(t) \rightarrow \phi K_s$ also applies to the decay $B_s^0(t) \rightarrow \phi(1680) K_s$, where $\phi(1680)$ is a radially excited ϕ . We expect the branching ratio for $B_s^0 \rightarrow \phi(1680) K_s$ to be almost a factor 10 larger than $B_s^0 \rightarrow \phi K_s$ [19]. This is because the form factor for $B_s^0 \rightarrow \phi$ (or, in general, for any $B \rightarrow$ light meson) probes the high-momentum tail of the ϕ wavefunction. As the radially excited $\phi(1680)$ has more high-momentum components, the form factor for $B_s^0 \rightarrow \phi(1680)$ is enhanced relative to $B_s^0 \rightarrow \phi$.

Note that the measurement of CP violation in $B_d^0(t) \rightarrow \phi K_s$ probes β [Eq. (3)]. If the measurement of β as extracted in this mode disagrees with the measurement of

β from $B_d^0(t) \rightarrow J/\Psi K_s$, this will indicate the presence of new physics in the $b \rightarrow s$ penguin amplitude, i.e. in the $b \rightarrow s$ flavour-changing neutral current (FCNC) [20]. Similarly, the value of β extracted in $B_s^0(t) \rightarrow \phi K_s$ or $B_s^0(t) \rightarrow \phi(1680)K_s$ can be compared with that found in $B_d^0(t) \rightarrow J/\Psi K_s$. Assuming that the weak phase of B_s^0 - \overline{B}_s^0 mixing is tiny — and this can be tested by looking for CP violation in $B_s^0(t) \rightarrow J/\Psi \phi$, for example — a discrepancy between these two values points clearly to new physics in the $b \rightarrow d$ FCNC. This new physics might affect B_d^0 - \overline{B}_d^0 mixing and/or the $b \rightarrow d$ penguin amplitude. Now, it is quite likely that CP violation in B_s^0 decays can only be measured at hadron colliders, since one needs an extremely large boost in order to resolve the rapid B_s^0 - \overline{B}_s^0 oscillations. Since hadron colliders produce copious amounts of B_d^0 and B_s^0 mesons, it should be possible to perform the $B_d^0(t) \rightarrow \phi K_s$ and $B_s^0(t) \rightarrow \phi K_s$ analyses simultaneously, since the final state is the same. Thus, by measuring β in these decay modes, at hadron colliders one can test for the presence of new physics in both the $b \rightarrow s$ and $b \rightarrow d$ FCNC's.

To summarize: in general, $b \rightarrow d$ penguin decays receive contributions from internal u , c and t quarks. Because of this, one cannot cleanly extract information about weak phases from measurements of CP-violating rate asymmetries. In this paper, we have shown that this does not hold for the decay $B_s^0(t) \rightarrow \phi K_s$. Due to a fortuitous cancellation between the $(V - A) \otimes (V - A)$ and $(V - A) \otimes (V + A)$ matrix elements, the contributions from the u - and c -quark operators each vanish. Thus, $B_s^0(t) \rightarrow \phi K_s$ is dominated by a single decay amplitude, that of the internal t -quark, so that the indirect CP asymmetry measures $\sin 2\beta$. This is the only $b \rightarrow d$ penguin decay for which this occurs.

Of course, this cancellation is not rigorous: it depends on our choosing particular values for the s - and b -quark masses. However, we have shown that, over most ($\sim 80\%$) of the theoretical parameter space, the difference between the indirect CP asymmetry and the true value of $\sin 2\beta$ is less than 10%. Furthermore, by using information about the direct CP asymmetry, one can get a better handle on the probable error on $\sin 2\beta$. For example, should a_{dir}^{CP} be found to be tiny, this increases the likelihood that $\sin 2\beta$ is being extracted with a small error: for $a_{dir}^{CP} \simeq 0$, the error on $\sin 2\beta$ is at most 5% over almost 90% of the parameter space.

Because the u - and c -quark contributions to $B_s^0(t) \rightarrow \phi K_s$ are very small, one can expect that the branching ratio for this decay will be reduced. This is indeed what is found: we estimate that $BR[B_s^0 \rightarrow \phi K_s] \sim 10^{-7}$. However, the same analysis also applies to the decay $B_s^0(t) \rightarrow \phi(1680)K_s$, where $\phi(1680)$ is a radially excited ϕ . The branching ratio for this decay is expected to be about 10 times larger than that for $B_s^0(t) \rightarrow \phi K_s$.

Finally, we note that $B_s^0(t) \rightarrow \phi K_s$ will probably be studied at a hadron collider. These same experiments can simultaneously measure the CP asymmetry in $B_d^0(t) \rightarrow \phi K_s$, which also probes $\sin 2\beta$ in the SM. By comparing the CP asymmetries in these two modes with that measured in $B_d^0(t) \rightarrow J/\psi K_s$, one can look for the presence of new physics in both $b \rightarrow d$ and $b \rightarrow s$ transitions.

Acknowledgments

C.S.K. thanks D.L. for the hospitality of the Université de Montréal, where some of this work was done. The work of C.S.K. was supported in part by the BK21 Program, SRC Program and Grant No. 2000-1-11100-003-1 of the KOSEF, and in part by the KRF Grants, Project No. 2000-015-DP0077. The work of A.D. and D.L. was financially supported by NSERC of Canada.

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